INSTITUTE of SPRING TECHNOLOGY

Tech-Spring Report 20B:
Non Axial Resonances in Compression Springs

1. Introduction

When designing a spring for a dynamic application only the axial resonance is considered. However a spring has six basic degrees of freedom and each has a fundamental resonant frequency and associated sub-harmonics.

Looking at the picture below it can be seen these are the linear movements in the axial direction X and the lateral movements in the Y and Z directions and torsional movements about this axis as defined by θx, θy, θz.

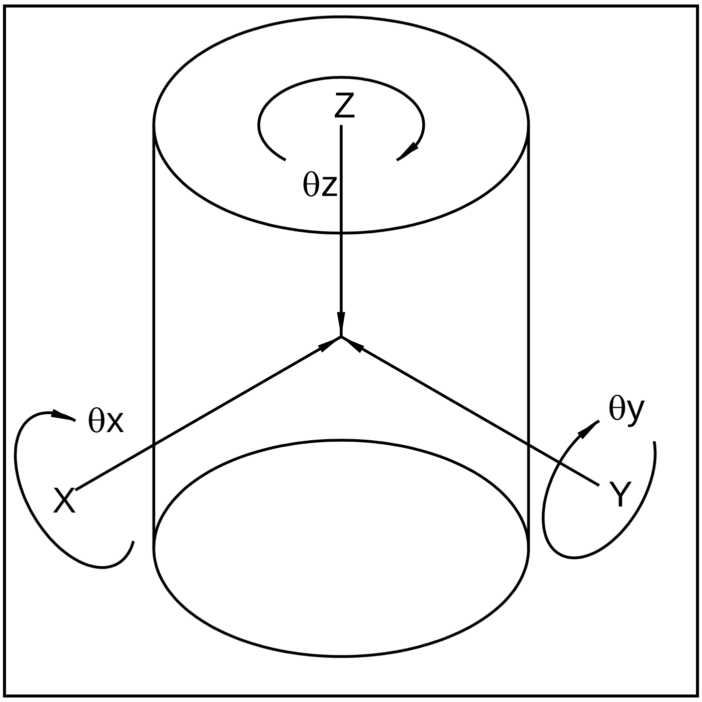


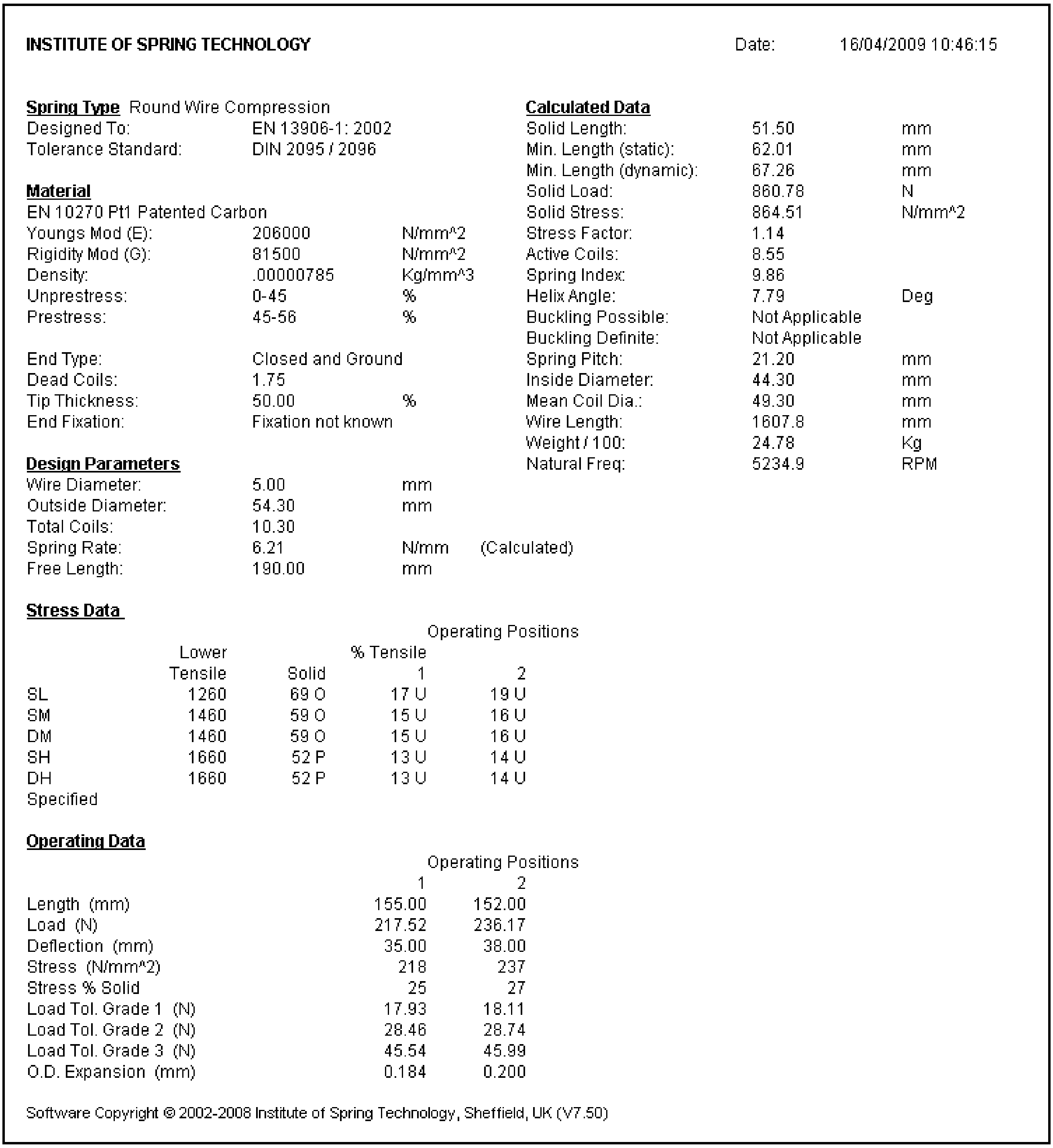
Fig 1 degrees of freedom

The geometry of a spring dictates that all the degrees of freedom are interlinked and it is not possible to deflect the spring in one axis without imparting some degree of deflection into the other deflection modes. It is widely known that as a compression spring is deflected the end coils try to rotate relatively to each other about the Z-axis, in the θz direction. Similarly all springs will bow when compressed or 'S' producing movements in the X and Y directions and/or θx and θy.

Thus if a spring is compressed dynamically at a frequency equal to the natural frequency of one of the other degrees of freedom then resonance will occur and manifest itself by large deflections being induced about that axis.

2. Equipment used

A spring to the following design (see Technical report 20A) was fitted with a strain gauge and installed into a fatigue-testing machine. The strain gauge equipment enabled monitoring and analysis of the vibrations in the spring.



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Spring Type Round Wire Compression

Designed To:	EN 13906-1: 2002		
Tolerance Standard:	DIN 2095 / 2096		

Material

EN 10270 Pt1 Patented Carbon

Youngs Mod (E):	206000	N/mm^2	
Rigidity Mod (G):	81500	N/mm^2	
Density:	.00000785	Kg/mm^3	
Unprestress:	0-45	%	
Prestress:	45-56	%	
End Type:	Closed and Ground		
Dead Coils:	1.75		
Tip Thickness:	50.00	%	
End Fixation:	Fixation not known		

Design Parameters

Wire Diameter:	5.00	mm	
Outside Diameter:	54.30	mm	
Total Coils:	10.30		
Spring Rate:	6.21	N/mm	(Calculated)
Free Length:	190.00	mm	

Calculated Data

Solid Length:	51.50	mm
Min. Length (static):	62.01	mm
Min. Length (dynamic):	67.26	mm
Solid Load:	860.78	N
Solid Stress:	864.51	N/mm^2
Stress Factor:	1.14	
Active Coils:	8.55	
Spring Index:	9.86	
Helix Angle:	7.79	Deg
Buckling Possible:	Not Applicable	
Buckling Definite:	Not Applicable	
Spring Pitch:	21.20	mm
Inside Diameter:	44.30	mm
Mean Coil Dia.:	49.30	mm
Wire Length:	1607.8	mm
Weight / 100:	24.78	Kg
Natural Freq:	5234.9	RPM

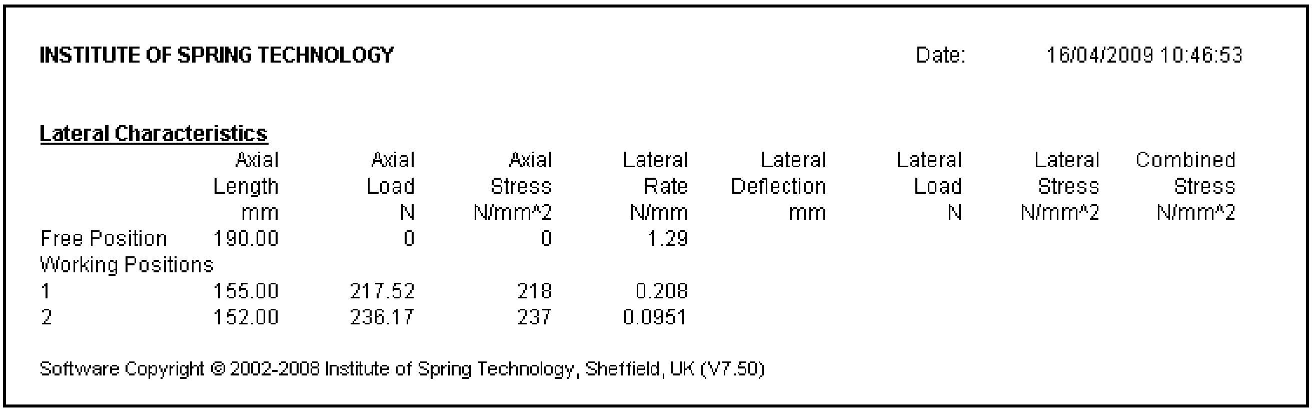
Stress Data

			Operating Positions	
	Lower Tensile	Solid	% Tensile 1	2
SL	1260	69 O	17 U	19 U
SM	1460	59 O	15 U	16 U
DM	1460	59 O	15 U	16 U
SH	1660	52 P	13 U	14 U
DH	1660	52 P	13 U	14 U
Specified				

Operating Data

	Operating Positions 1	2
Length (mm)	155.00	152.00
Load (N)	217.52	236.17
Deflection (mm)	35.00	38.00
Stress (N/mm^2)	218	237
Stress % Solid	25	27
Load Tol. Grade 1 (N)	17.93	18.11
Load Tol. Grade 2 (N)	28.46	28.74
Load Tol. Grade 3 (N)	45.54	45.99
O.D. Expansion (mm)	0.184	0.200

Fig 2. Axial Characteristics of Test Spring



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Lateral Characteristics
                    Axial       Axial       Axial     Lateral     Lateral     Lateral     Lateral   Combined
                   Length        Load      Stress        Rate  Deflection        Load      Stress     Stress
                       mm           N     N/mm^2        N/mm          mm           N     N/mm^2     N/mm^2
Free Position      190.00           0           0        1.29
Working Positions
1                  155.00      217.52         218       0.208
2                  152.00      236.17         237      0.0951

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Fig 3. Lateral characteristics of test spring.

3. Theoretical calculation of the torsional and lateral natural frequency

3.1 Torsional natural frequency

To calculate the torsional natural frequency, the compression spring is considered as a torsion spring with the end coils removed. It is assumed that only the ground ends are inactive which equates to *Total coils – 1.5*.

The formula for a torsion spring with both ends fixed is:

$$\omega_{nTor} = \frac{d}{4\pi D^2 n}\sqrt{\frac{E}{\rho}}$$

Where:
ω_n=Natural frequency
E = Young's modulas (2.06x10^11N/M^2)
d = Wire diameter
D = Mean coil diameter
ρ = Density (7850 kg/M^3)
n = Number of active turns

Note: Alwaysuse M,Kg,N units

This calculates out to: ω_{nTor} = 95.3Hz

3.2 Lateral natural frequency

By analogy a compression spring laterally vibrating about its centre is similar to a beam, built in at both ends and of constant mass/length.

The formula for such a beam is:

$\omega_n = 3.56\sqrt{\frac{E \times I}{Mass \times L^3}}$ or $\omega_n = \sqrt{\frac{12.67 \times E \times I}{Mass \times L^3}}$ which must be the general form $\omega_n = \sqrt{\frac{k}{Mass}}$

And hence $k_{\omega n} = \frac{12.67 \times E \times I}{L^3}$

The centre stiffness *k* for a built in beam is: $k = \frac{192 \times E \times I}{L^3}$

Thus the static stiffness in the beam $k_{beam} = \frac{192 \times E \times I}{L^3}$ must be equivalent to the stiffness used for calculating the natural frequency $k_{\omega n} = \frac{12.67 \times E \times I}{L^3}$

So to use the beam centre stiffness it must be factored $k = \frac{12.7 \times k_{beam}}{192}$

So $\omega_n = \sqrt{\frac{k}{15.12 \times Mass}}$ or $\omega_n = \frac{1}{3.89}\sqrt{\frac{k}{Mass}}$ where *k* = Lateral stiffness of the beam

Thus we can use this formula to calculate the natural frequency of a spring vibrating about the lateral centre of a spring.

If the spring is considered as two half springs oscillating in unison, the analysis only needs to consider a single half and calculate its lateral spring rate.

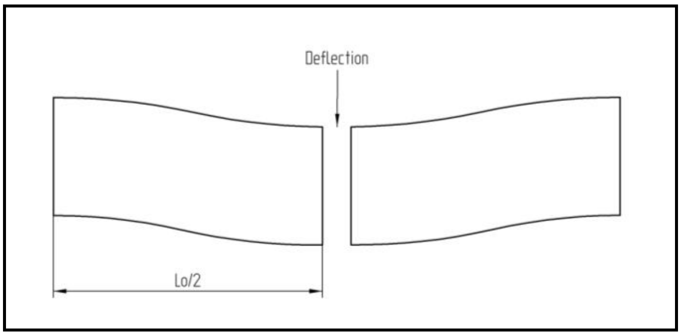

Fig 4. Halving of test spring for theoretical analysis.

For this IST's version 7.5 cad software was used. Care has to be taken in the estimation of the active coils and hence the active mass of the spring.

Lateral deflection will induce shearing into the dead end coil, thus more coil will be active than in the axial mode, and hence a dead end coil factor of 1.5 has been used.

Thus the half spring will have a total number of turns:

$$N_{half} = \frac{N_{total} - 1.5}{2} + 1.5$$

And an active number of turns:

$$n_{0.} = \frac{N_{total} - 1.5}{2}$$

For the test spring:

N_{half} = 5.9
n_{half} = 4.4

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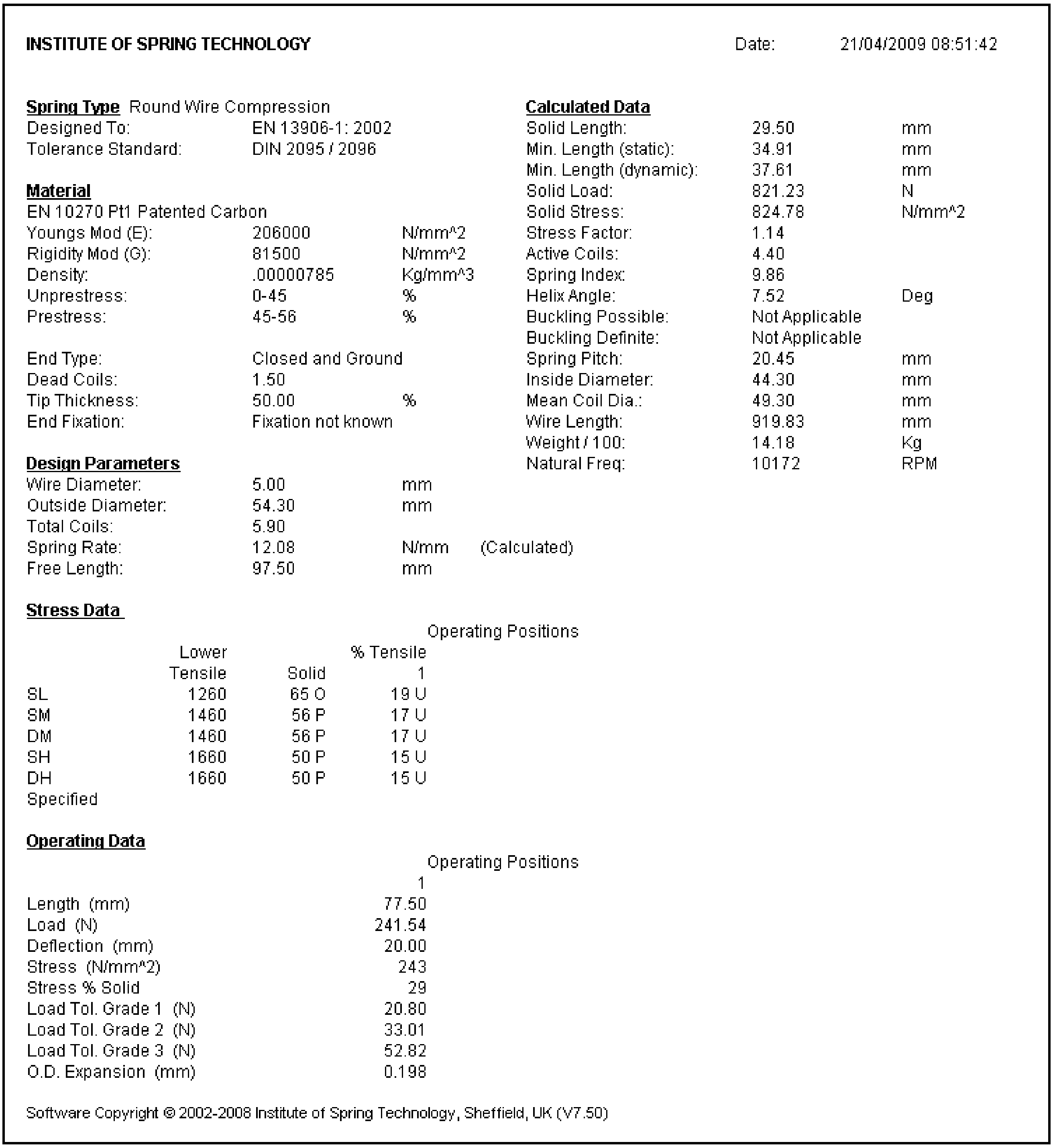

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Spring Type  Round Wire Compression                    Calculated Data
Designed To:           EN 13906-1: 2002                Solid Length:               29.50        mm
Tolerance Standard:    DIN 2095 / 2096                 Min. Length (static):       34.91        mm
                                                       Min. Length (dynamic):      37.61        mm
Material                                               Solid Load:                 821.23       N
EN 10270 Pt1 Patented Carbon                           Solid Stress:               824.78       N/mm^2
Youngs Mod (E):        206000          N/mm^2          Stress Factor:              1.14
Rigidity Mod (G):      81500           N/mm^2          Active Coils:               4.40
Density:               .00000785       Kg/mm^3         Spring Index:               9.86
Unprestress:           0-45            %               Helix Angle:                7.52         Deg
Prestress:             45-56           %               Buckling Possible:          Not Applicable
                                                       Buckling Definite:          Not Applicable
End Type:              Closed and Ground               Spring Pitch:               20.45        mm
Dead Coils:            1.50                            Inside Diameter:            44.30        mm
Tip Thickness:         50.00           %               Mean Coil Dia.:             49.30        mm
End Fixation:          Fixation not known              Wire Length:                919.83       mm
                                                       Weight / 100:               14.18        Kg
Design Parameters                                      Natural Freq:               10172        RPM
Wire Diameter:         5.00            mm
Outside Diameter:      54.30           mm
Total Coils:           5.90
Spring Rate:           12.08           N/mm     (Calculated)
Free Length:           97.50           mm

Stress Data
                                            Operating Positions
              Lower                  % Tensile
            Tensile       Solid              1
SL             1260        65 O           19 U
SM             1460        56 P           17 U
DM             1460        56 P           17 U
SH             1660        50 P           15 U
DH             1660        50 P           15 U
Specified

Operating Data
                                            Operating Positions
                                                 1
Length  (mm)                                 77.50
Load  (N)                                   241.54
Deflection  (mm)                             20.00
Stress  (N/mm^2)                               243
Stress % Solid                                  29
Load Tol. Grade 1  (N)                       20.80
Load Tol. Grade 2  (N)                       33.01
Load Tol. Grade 3  (N)                       52.82
O.D. Expansion  (mm)                         0.198

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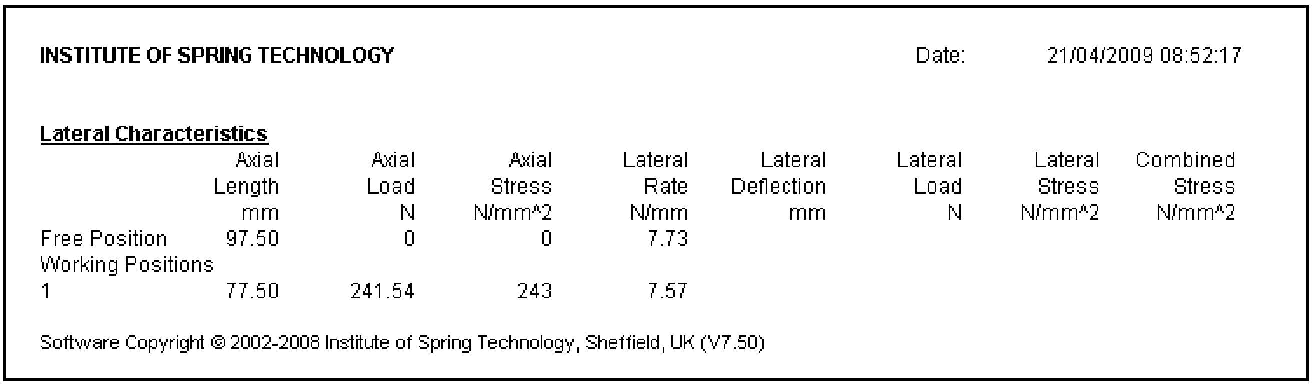
Fig 5. Axial Characteristics of Half Test Spring.



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Lateral Characteristics
                    Axial       Axial       Axial     Lateral      Lateral     Lateral     Lateral    Combined
                   Length        Load      Stress        Rate   Deflection        Load      Stress      Stress
                       mm           N     N/mm^2        N/mm           mm           N      N/mm^2      N/mm^2
Free Position       97.50           0           0        7.73
Working Positions
1                   77.50      241.54         243        7.57

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Fig 6. Lateral characteristics of half test spring.

It can be seen that the lateral spring rate is 7570N/m at the test positions.
The mass of the active coils can be found by:

$$m = Springweight \times \frac{activecoils}{totalcoils}$$

m = 0.106kg

Using the formula:

$$\omega_{nLat} = \frac{1}{3.89}\sqrt{\frac{k}{m}}$$

ω_{nLat} = Natural frequency
k = Lateral spring rate
m = mass of active coils

Note: Always use M,Kg,N units

This calculates out to: ω_{nLat} = 70.5Hz

4. Test programs

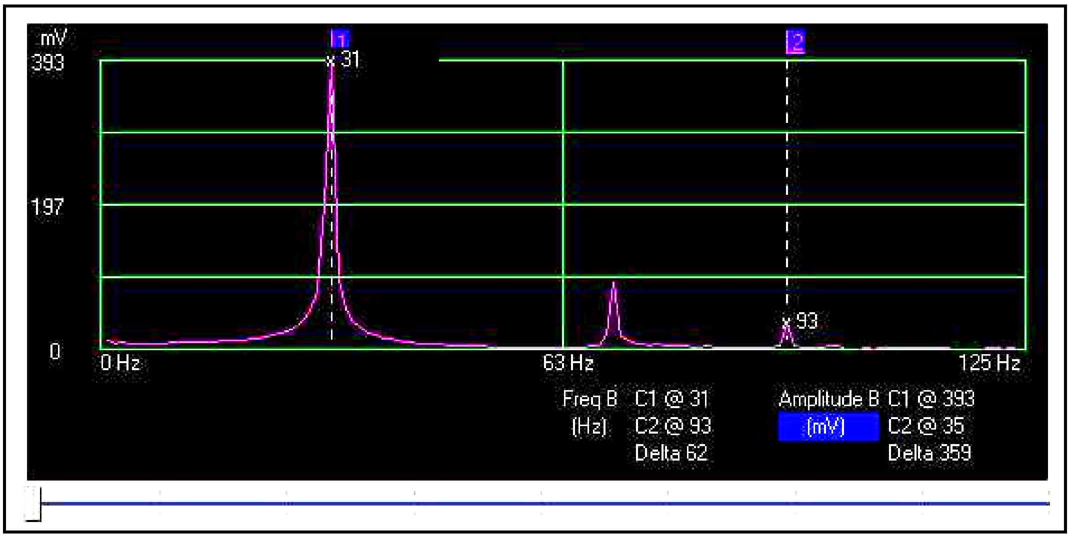
Two programs of work were initiated, the first used a strain gauge to monitor for resonant peaks, and the second used a special natural frequency test machine to measure the lateral natural frequency.

4.1 Strain gauge tests

The test speed was slowly increased from 4Hz up to 70Hz looking for resonance peaks that occurred at sub-harmonics of the calculated torsional and lateral natural frequencies. The FFT screen was captured and can be seen below.

4.1.1 Torsional resonance test results

For the torsional resonance it can be seen that at test speeds of 31Hz and 47 Hz a resonance peak occurred at 93Hz, which is close to the calculated ω_n of 95.3Hz. As expected the 2nd sub harmonic produced a much higher peak than the 3rd.



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Freq B  C1 @ 31     Amplitude B  C1 @ 393
(Hz)    C2 @ 93     (mV)         C2 @ 35
        Delta 62                 Delta 359
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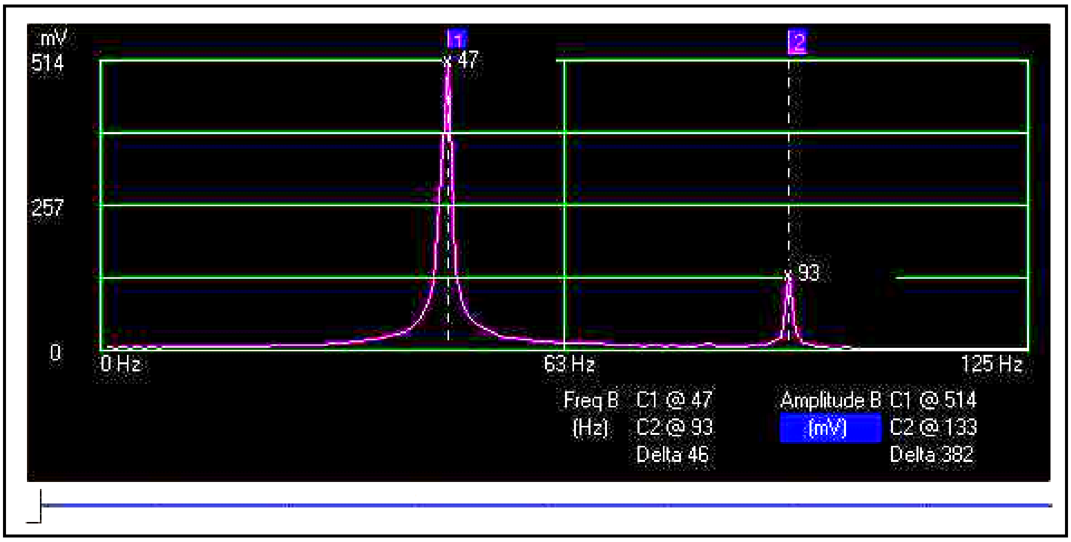
Fig 7. FFT output for torsional resonance at 3rd sub-harmonic



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Freq B  C1 @ 47     Amplitude B  C1 @ 514
(Hz)    C2 @ 93     (mV)         C2 @ 133
        Delta 46                 Delta 382
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Fig 8. FFT output for torsional resonance at 2nd sub-harmonic

4.1.2 Lateral resonance test results

For the lateral tests it can be seen that resonance occurred at sub-harmonics of 23Hz and 35Hz.

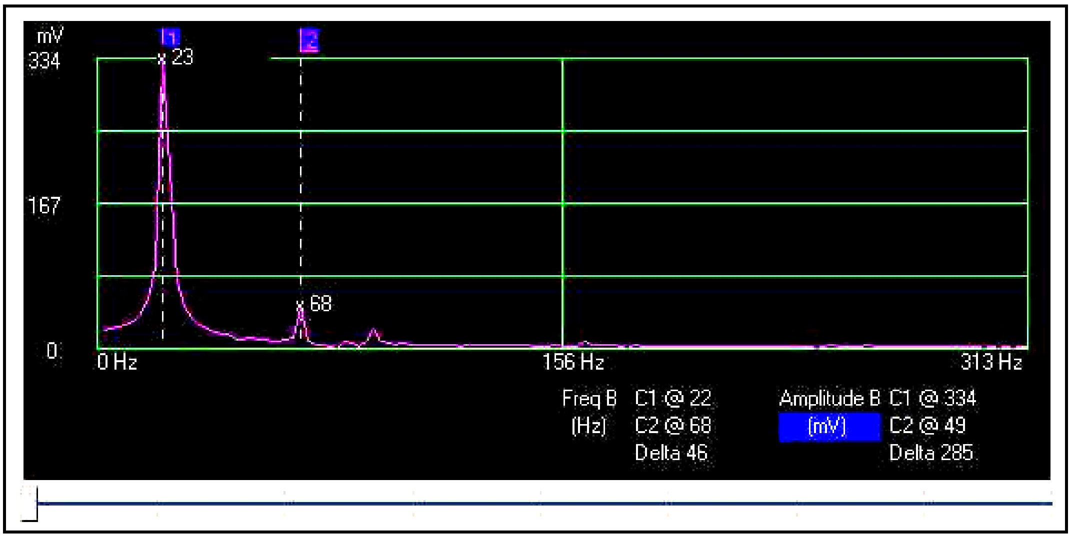

Fig 9. FFT output for Lateral resonance at 3rd sub-harmonic

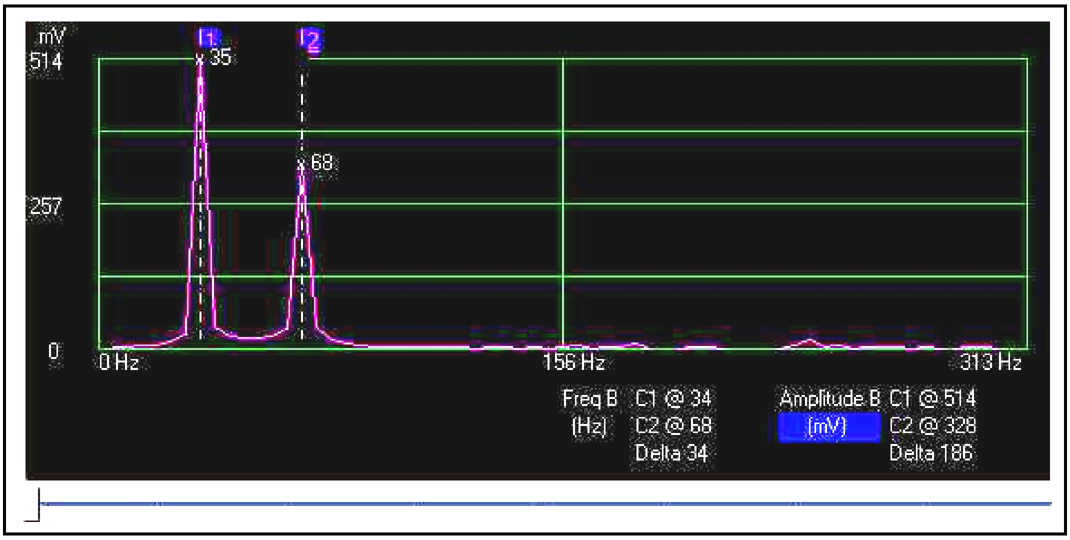

Fig 10. FFT output for Lateral resonance at 2nd sub-harmonic

4.2 Lateral natural frequency measurements

This test comprises of deflecting the spring in the middle and releasing to set up a lateral vibration. A sensor is used to monitor this and record the resulting data.

Test spring	Wire diam (mm)	OD (mm)	Total coils	Lo (mm)	Test length (mm
1	5	54.3	10.3	190	155
2	2	23.35	12.75	87.5	86
3	4	33.03	6.9	53.4	51
4	2.35	31.5	5.5	74.3	56

Test spring	Calculated Lateral Frequency (Hz)	Measured Lateral Frequency (Hz)	% error
1	70	68	+2.8
2	114	113	+1
3	388	379	+2.3
4	282	278	+1.4

5. Discussion

From the above results it can be seen that the torsional and lateral natural frequencies can be calculated with reasonable precision. However the lateral rate calculations are very sensitive to the active number of coils and the associated active coil mass. With a spring of few coils the estimation of active coils will be highly subjective, however it appears that using a dead end coil factor of 1.50 is reasonable for both the torsional and lateral calculations. Extending the theoretical length of the half spring by half a wire diameter to account for the end coil tip, which does not exist in the real spring will also improve the lateral rate calculation accuracy.

Springs that have gentle end coil layon will see an increase in spring rate and a reduction in active mass, which will combine to give and increase in natural frequency as deflection is increased.

Springs with high helix angles and indexes will see significant increase in coil diameter during compression. This can combine to give a reducing rate and hence natural frequency as the spring is compressed. High helix angles can also produce considerable inaccuracies in the lateral spring rate calculations.

The relative effect of these non-axial resonances will depend on the actual shape of the spring. Springs with high helix angles will produce a relatively large torsional movement and hence would be expected to produce a much higher resonance response that a low helix angle spring.

With lateral resonance it needs to be remembered that the lateral rate is not a constant and changes with deflection. The amount of change is dependant on the spring design but it can be considerable.

As a guide, long slender springs will have a low lateral rate and corresponding natural frequency. Their lateral oscillations will also be higher.

The effect on stress in the spring can be assumed to be detrimental, but it is outside of the scope of this report to quantify the effect.

6. Conclusions

1. A spring has more than one mode of resonance and these frequencies should be considered when designing a spring for a dynamic application.
2. The lateral and torsional natural frequencies can be readily calculated.
3. The theoretical and practical test results gave reasonable correlation.
4. Springs with gentle end coil layon will produce a rising spring rate and hence natural frequency with compression.
5. Compression springs do not have constant lateral rates and hence the lateral natural frequency will vary depending on the nominal compression.
6. Springs with high helix angles and indexes may exhibit a reducing natural frequency due to the reduction in spring rate caused by coil diameter growth.
7. Springs with high helix angles and indexes are subject to inaccuracies in the calculation of lateral rate and thus theoretical frequencies will be subject to the same inaccuracies.
8. Any additional movements of coils due to non-axial resonance can be assumed to be detrimental to spring life.